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Differential Game of Two Pursuers Chasing One Evader with Different Forms of Constraints in $l_2 -$ Space

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Abstract

This paper addresses a pursuit differential game involving two Pursuers chasing one Evader within the l_2-space for an infinite system of first-order differential equations. The first Pursuer employs a strategy that satisfied integral constraint, while the second Pursuer uses a strategy governed by a geometric constraint. The goal of each pursuer is to force the state of the system to coincide with a predefined state within a finite time, counteracting the Evader's opposing actions. We construct an explicit strategy to determine the conditions necessary for successful pursuit. Moreover, we explore a control problem involving a single player.

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1.0 Introduction

There has been significant research dedicated to differential games where the players' control functions are subject to either integral or geometric constraints. For instance, numerous studies such as those referenced in Idham *et al*., (2016), Samatov *et al*., (2013), Ibragimov *et al*., (2017), Ibragimov (2004), Mamatov (2008), Ibragimov (2013), Ibragimov *et al*., (2015), Jafaru and Ibragimov (2012), Salimi *et al*., (2016), Gafurjan and Mehdi (2009), Ibragimov *et al*., (2017), Waziri, *et al*., (2022), and Usman *et al*., (2021), have garnered the attention of many researchers.

According to Idham *et al*., (2016) studied a simple motion differential game of many pursuers and one evader in the plane. Control functions of players are subjected to coordinate-wise integral constraints.

In the paper of Alias *et al*., (2017), considered an evasion differential game of infinitely many evaders from infinitely many pursuers was studied in Hilbert space l_2 and evasion strategies were constructed in explicit form. They proved that if the state of the evader y, coincides with that of a pursuer $x_i, i = 1,2, \dots, m,$ at some time, then pursuit is completed. An equation for the completing of pursuit is obtained from any position of the players in the given set. Moreover, strategies for the pursuers is constructed. In the paper of Usman *et al*., (2022), investigated a differential game of pursuit for an infinite system of simple motion in a plane. The control functions of the players satisfy both geometric and integral constraints respectively. In the plane, the game assumes to be completed if the state of the pursuer is directly coincides with that of the evader at some time and the evader tries to stop the incidence. In addition to that the strategy of the pursuer with respect to geometric and integral constraints will be constructed. Moreover, a numerical example will be given to illustrate the result.

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In the paper of Ibragimov (2004), a pursuit differential game of m pursuers and k evaders with integral constraints described by the systems of differential equations of equation (2) above with the condition that $i = 1,2,\dots,m$ and $j = 1,2,\dots,k$. Control functions of the is satisfied integral constraints. Under assumption that the total resource of controls of the pursuers is greater than that of the evaders. It was proved that pursuit can be completed from any initial position.

Several research works have focused on differential games involving the transfer of a system's state with distributed parameters from one stage to another, particularly to the origin (see, for instance, Tukhtasinov and Mamatov (2009), Tukhtasinov and Mamatov (2008), Chernous'ko (1992), Satimov and Tukhtasinov (2005), and Mamatov and Tukhtasinov (2009).

In paper, for Tukhtasinov (1995), a pursuit game problem is proposed within a control system of distributed parameters for transferring the state of the system to the zero state (i.e., the origin) in the presence of a player attempting to prevent this. The game is modeled by partial differential equations, where the right-hand side, containing the player's control, is in additive form.

$$\frac{\partial z}{\partial t} + Az(\Omega) = u(\Omega) + v(\Omega), z(0) = z_0, 0 < t < \Omega,$$

where

$$Az = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial z}{\partial x_i}\right),$$

is a linear operator independent of t. Decomposition method is also applied to reduce the problem to an infinite system of ordinary differential equations.

$$\dot{z_k}(\Omega) + \lambda_k z_k(\Omega) = u_k(\Omega) + v_k(t), \qquad (2)$$

with $z_k(0) = z_{k0}, 0 < t < \Omega, k = 1,2,\dots,$

where $\lambda_k, k = 1,2,\dots$ are eigenvalue of the operator. A Satisfying $0 \le \lambda_1 \le \lambda_2, \dots \to \infty$. The game involved the integral, geometric and both constraints on the player's control functions. Conditions which are sufficient for completion of the game is also presented.

Differential game pursuit time was obtained by Ibragimov *et al.*, (2017) for the game of two players with different target described by system of infinite differential equations (2) in the space l_2, when the controls functions of the players are subjected to geometric constraints. The target of the first player is to bring the state of the system towards the origin against the actions of the second player tries avoid this. In the game, the strategy of the first player happen to be a Pursuer is constructed in an explicit form. Therefore, the present paper proposes to study a pursuit differential game problem of two pursuers chasing one evader with different forms of constraints for an infinite system of first-order differential equations described as:

$$\dot{z_{ik}}(\Omega) + \lambda_{ik} z_{ik}(\Omega) = u_{ik}(\Omega) + v_k(\Omega), \qquad (3)$$

with

$$z_{ik}(0) = {z^0}_{ik}, 0 < t < \Omega, k = 1,2,\dots, i = 1,2$$

where $z_{ik}, u_{ik}, v_k \epsilon \mathbb{R}, k = 1,2,\dots. i = 1,2,$ and $z^0 = ({z^0}_1, {z^0}_2, \dots) \epsilon\, l_2, u_{i1}, u_{i2}, \dots, \& v_1, v_2, \dots,$ are control parameters of the two pursuers and the a evader respectively. $\lambda_k, k = 1,2,\dots$ is non negative real numbers, in Hilbert space l_2, where

$$l_2 = \left\{\mathfrak{z} = (\mathfrak{z}_1, \mathfrak{z}_2, \dots) \middle| \sum_{k=1}^{n} |\mathfrak{z}_k|^2 < \infty\right\},$$

with

$$\langle \mathfrak{z}, X\rangle = \sum_{k=1}^{n} \mathfrak{z}_k X_k, \mathfrak{z}, X \epsilon\, l_2, \qquad |\mathfrak{z}||^2 = \sum_{k=1}^{n} |\mathfrak{z}_k|^2,$$

be an inner product and norm respectively. The pursuer attempts to bring the state of the system to coincide with another state and the evader actions in the opposite. In contrast of previous studies of differential games Usman *et al.*, (2019) and Gafurjan *et al.*, (2018) that dealt with the case of only single Pursuer.

This paper is organized as follows. The second section present the formulation of the problems and some important definitions which will be required in the subsequence sections. In the third section, present a control function problem of the single players. Differential game of two pursuers by one evader with different Forms of Constraints is considered in section four. In the last but not the least section, we give the concluding part of the paper.

2.0 Formulation of the Problem

Let

$$L_2(0,\Omega,l_2) = \left\{w = (w_1, w_2, \dots) \middle| \int_0^\Omega ||\mathfrak{z}(t)||^2 dt < \infty, w_1(.) \epsilon\, L_2(0,\Omega)\right\},$$

where

$$||\mathfrak{z}(t)||^2 = \sum_{k=1}^{\infty} |\mathfrak{z}_k(t)|^2, t \epsilon [0,\Omega].$$

Let define $z^1 = ({z^1}_{i1}, {z^1}_{i2}, \dots) \epsilon\, l_2,\ i = 1,2,$ be a given another state far away from initial state.

Definition 2.1 *A function* $w(\cdot) = (w_1(\cdot), w_2(\cdot), \ldots) \epsilon\, l_2$ *such that* $w(\cdot): [0, \Omega] \to l_2$ *With measurable coordinate* $w_k(t), k = 1,2, \ldots, 0 < t < \Omega$, *satisfy the conditions*

$$\|w(t)\| = \left(\sum_{k=1}^{\infty} |w_k(t)|^2\right)^{1/2} \leq \propto,$$

and

$$\left(\int_0^{\Omega} ||w(t)||^2 dt\right)^{1/2} \leq \propto, t\, \epsilon\, [0, \Omega],$$

then the function $w(\cdot)$ *is called an admissible control subject to geometric and integral constraints respectively, where* $\propto$ *is a nonnegative number.*

Let $S(\propto)$ be the set of all admissible controls with respect to geometric constraint and $S^*(\propto)$ be that of integral constraint.

Definition 2.2 *The control functions* $u(\cdot)$ *and* $v(\cdot)$ *of the pursuers and the evader are said to be admissible if it's satisfies one of the following conditions:*

$$||u_i(t)||^2 = \sum_{k=1}^{\infty} |u_{ik}(t)|^2 \leq {\rho_i}^2, ||v(t)||^2 = \sum_{k=1}^{\infty} |v_k(t)|^2 \leq \sigma^2, \quad (4)$$

$$\int_0^{\Omega} ||u_i(t)||^2 dt \leq {\rho_i}^2, \quad \int_0^{\Omega} ||v(t)||^2 dt \leq \sigma^2. \quad (5)$$

The Infinite system (3) in which $u(\cdot) = (u_1(\cdot), u_2(\cdot), \ldots) \epsilon\, l_2$ *and* $v(\cdot) = (v(\cdot), v_2(\cdot), \ldots) \epsilon\, l_2$ *satisfy inequalities (4) and (5) respectively, are called differential games* Φ_1 *and* Φ_2 *respectively.*

Definition 2.3 *Let* $w(\cdot)$*belongs to* $S(\rho)$ *and* $w(\cdot)$*belongs to* $S^*(\rho)$, *then one will say that a function* $z(\Omega) = (z_1(\Omega), z_2(\Omega), \ldots), 0 < t < \Omega$ *with* $z_k(0) = {z^0}_k, k = 1,2, \ldots$ *is called solution of the initial value problem*

$$\dot{z_k}(\Omega) + \lambda_k z_k(\Omega) = w_k(\Omega), z_k(0) = {z^0}_k, 0 < t < \Omega, k = 1,2, \ldots, \quad (6)$$

If $z_k(\Omega), k = 1,2, \ldots$ *are absolutely continuous and almost everywhere on* $[0, \Omega]$*satisfy equation (6).*

Assume that the space $C(0, \Omega, l_2)$ of the continuous function $z(\cdot) = (z_1(\cdot), z_2(\cdot), \ldots), 0 < t < \Omega$ such that for each $t, 0 < t < \Omega$, $z(\Omega)$ belongs l_2.

Definition 2.4 *Let* $u_i(t, v), i = 1,2$, *be the given functions such that* $u: [t, v] x l_2 \to l_2$ *of the form*

$$u_{ik}(t, v) = v_k + w_k \; k = 1,2, \ldots$$

Is called the strategy of the pursuer with respect to geometric constraint if:

i. *For any control* $v(\cdot)$ *of the evader (i.e., the control* $v(\cdot)$ *is admissible), the system (3) has the only solution at* $u_k(t) = u(t, v)$, *where* $w(\cdot) = (w_1(\cdot), w_2(\cdot), \ldots) \epsilon\, S(\rho - \sigma)$.
ii. $u(\cdot) = (\cdot, v(\cdot)) \epsilon\, S(\rho)$.

In similar passion, one will define the pursuer's strategy with respect to integral constraint and $u(\cdot) = (\cdot, v(\cdot)) \epsilon\, S^*(\rho)$.

Definition 2.5 *If there exists a pursuer's strategy* $u(t, v)$ *such that for any control of the evader* $v(\cdot)$ *(i.e., the control* $v(\cdot)$ *is admissible) to ensure and guaranteed* $z_k(t) = {z_k}^1, k = 1,2, \ldots$ *at some* $t, 0 < t < \Omega$, *then one will say that pursuit can be completed, in the differential game* Φ_1 *and* Φ_2 *respectively, where* $z_k(\Omega)$*is the solution of the system (3).*

It has been proven in [18] that if $w(\cdot)$belong to $S(\rho)$ and $\delta_k > 0$, then for any given $\Omega > 0$ in the space $C(0, \Omega, l_2)$,the infinite system (6) has a unique solution $z(\Omega) = (z_1(\Omega), z_2(\Omega), \ldots), 0 < t < \Omega$, where $z_k(\Omega), 0 < t < \Omega, k = 1,2, \ldots$ defined by

$$z_k(\Omega) = e^{\delta_k \Omega} \beta_k(\Omega), k = 1,2, \ldots, \quad (7)$$

with

$$\beta_k(\Omega) = {z_k}^0 + \int_0^{\Omega} e^{\delta_k s} w(s) ds, k = 1,2, \ldots,$$

and $\lambda_k = \delta_k > 0$.

The Problem is to find a condition: For which pursuit can be completed in the differential game Φ_1 and Φ_2.

3.0 Control Problem of Single Player:

In this part, we study a control problem for single payer of an infinite systems (6).

Now, for the first problem, we define the following set:

$$\gamma_1(\Omega) = \left\{(z^0, z^1) \middle| \sum_{k=1}^{n} (2|{z_k}^0|^2 q_k(\Omega) + +2|{z_k}^1|^2 {q_k}^*(\Omega)) \leq \propto^2 \right\},$$

where

$$q_k(\Omega) = \frac{\delta^2{}_k}{(e^{\delta_k \Omega} - 1)^2},\ q_k{}^*(\Omega) = -\,q_k(-\Omega), k = 1,2,\ldots$$

For second problem, we define the set as follows:

$$\gamma_2(\Omega) = \left\{(z^0, z^1) | \sum_{k=1}^{n}(2|z_k{}^0|^2 p_k(\Omega) + 2|z_k{}^1|^2 p_k{}^*(\Omega)) \leq \propto^2\right\},$$

where

$$p_k(\Omega) = \frac{2\delta_k}{e^{2\delta_k \Omega} - 1},\ p_k{}^*(\Omega) = -\,p_k(-\Omega), k = 1,2,\ldots$$

The first problem is find controls $w(\cdot) = (w_1(\cdot), w_2(\cdot), \ldots) \epsilon\, S(\rho)$ to forward that state of the system (6) from initial state into another state for finite time.

Lemma 3.1 *Let $z^0, z^1 \epsilon l_2$. If in general $z^0, z^1 \epsilon \gamma_1(\Omega)$, then there exist a control $w(\cdot) \epsilon\, S(\rho)$ to bring the state of the system (6) from initial state z^0 to coincide with another state z^1 for finite time.*

Proof: We consider a control problem for a first player satisfy the geometric constraint.
We construct the control as follows:

$$w_k(t) = [z_k{}^1 e^{\delta_k \Omega} - z_k{}^0]\ \sqrt{q_k(\Omega)}, \qquad (8)$$

where $0 < t < \Omega, \mathrm{k} = 1,2, \ldots$
Next to show that the defined control is admissible.
Using (8), we have

$$||w(t)||^2 = \sum_{k=1}^{\infty}|w_k(t)|^2 = \sum_{k=1}^{\infty}\left|\left[z^1 e^{\delta_k \Omega} - z^0\right]\sqrt{q_k(\Omega)}\right|^2 = \sum_{k=1}^{\infty}\left|z^1 e^{\delta_k \Omega} - z^0\right|^2 q_k(\Omega)$$

Using the obvious inequality
$|a-b|^2 = 2|a|^2 + 2|b|^2$.
It's not difficult that

$$||w(t)||^2 \leq \sum_{k=1}^{\infty}\left(2|z_k{}^1|^2 e^{2\delta_k \Omega} - 2|z_k{}^0|^2\right) q_k(\Omega) = 2\sum_{k=1}^{\infty}\left(|z_k{}^1|^2 q_k{}^*(\Omega) - |z_k{}^0|^2\, q_k(\Omega)\right) \leq \propto^2.$$

Consequently, (8) is admissible.
The last but not the least of the **lemma 3.1,** is prove that
$z_k(\Omega) = z_k{}^1, k = 1,2, \ldots,$
using equation (7) and admissible control (8), we get

$$\beta_k(\Omega) = z_k{}^0 + \int_0^{\Omega} e^{\delta_k s}\left(\left[z_k{}^1 e^{\delta_k \Omega} - z_k{}^0\right]\sqrt{q_k(\Omega)}\right) ds, = z_k{}^0 + \left(z_k{}^1 e^{\delta_k \Omega} - z_k{}^0\right) = z^1 e^{\delta_k \Omega}$$

Thus,
$z_k(\Omega) = \beta_k(\Omega) e^{-\delta_k \Omega} = z_k{}^1$
Consequently, the system (6) is transferred from $z^0$to z^1.
The proof of the first lemma 3.1 is completed.

For the second problem is to find control $w(\cdot) = (w_1(\cdot), w_2(\cdot), \ldots) \epsilon\, S^*(\rho)$ to forward the state of (6) from one state into another for a finite time.

Lemma 3.2 *Let $z^0, z^1 \epsilon \gamma_2(\Omega)$, then there exist a control $w(\cdot) \epsilon\, S^*(\rho)$ to bring the state of the system (6) from z^0 to coincide with z^1 for finite time.*

Proof: Define the control that $w(\cdot)$ satisfy the integral constraint

$$w_k(t) = e^{\delta_k t}[z_k{}^1 e^{\delta_k \Omega} - z_k{}^0]\ p_k(\Omega), \quad (9)$$

where $0 < t < \Omega, \mathrm{k} = 1,2, \ldots$
Show that control (9) is admissible. Using (9), we have

$$\int_0^{\Omega} ||w(s)||^2 ds = \sum_{k=1}^{\infty}\int_0^{\Omega} |e^{\delta_k s}[z_k{}^1 e^{\delta_k \Omega} - z_k{}^0] p_k(\Omega)\,|^2 ds = \sum_{k=1}^{\infty}|z_k{}^1 e^{\delta_k \Omega} - z_k{}^0|^2 (p_k(\Omega))^2 \int_0^{\Omega} e^{2\delta_k s} ds$$

As usual from obvious inequality $|a-b|^2 = |a|^2 + |b|^2$,
we obtain

$$\int_0^{\Omega} ||w(s)||^2 ds \leq \sum_{k=1}^{\infty}\left(2|z_k{}^1|^2 e^{2\delta_k \Omega} - 2|z_k{}^0|^2\right)(p_k(\Omega))^2 \int_0^{\Omega} e^{2\delta_k s} ds$$

$$= \sum_{k=1}^{n}(2|z_k{}^0|^2 p_k(\Omega) + 2|z_k{}^1|^2 p_k{}^*(\Omega)) \leq \propto^2.$$

Therefore, strategy (9) is admissible.
Next, is forwarding the state of the system to another.
Using (7) and constructed control (9), it's easy to see that

$$\beta_k(\Omega) = {z_k}^0 + \int_0^\Omega e^{\delta_k s}\left(e^{\delta_k s}\left[{z_k}^1 e^{\delta_k \Omega} - {z_k}^0\right]\ p_k(\Omega)\right) ds$$
$$= {z_k}^0 + \left({z_k}^1 e^{\delta_k \Omega} - {z_k}^0\right) = {z_k}^1 e^{\delta_k \Omega}.$$

indeed,

$z_k(\Omega) = \beta_k(\Omega) e^{-\delta_k \Omega} = {z_k}^1.$

Let

$\varphi_1(\Omega) = \{(z^0, z^1) |\ \sum_{k=1}^{n}(2|{z_{ik}}^0|^2 q_k(\Omega) + 2|{z_{ik}}^1|^2 {q_k}^*(\Omega)) \le (\rho_i - \sigma)^2, \}$, and

$\varphi_2(\Omega) = \left\{(z^0, z^1) \middle| \sum_{k=1}^{n}(2|{z_{ik}}^0|^2 p_k(\Omega) + 2|{z_{ik}}^1|^2 {p_k}^*(\Omega)\right) \le (\rho_i - \sigma)^2, \}$. Where $\rho_i > \sigma, i = 1,2, k = 1,2, \ldots$

Now, we proof the main result.

Theorem 4.1 *Let* $\rho_i > \sigma$ *and* $z^0, z^1 \in l_2$. *If in addition,* $z^0, z^1 \in (\varphi_1(\Omega),\ \varphi_2(\Omega))$, *then pursuit can be completed in differential game* (Φ_1, Φ_2) *for the time* Ω.

Proof: We proof the theorem in two different forms of constraints.

We construct the strategy for the first Pursuer on $[0, \Omega]$ that satisfy geometric constraint as Follows

Proof that strategy (10) and (11) are admissible. From the fact that $u_i(\cdot), i = 1$ belongs to $S^*(\cdot)$ and

$$\|u_i(t)\|^2 = \left(\sum_{k=1}^{\infty} |u_{ik}(t)|^2\right)^{\frac{1}{2}}$$
$$= \left(\sum_{k=1}^{\infty} \left|v_k(t) - \left[{z_{ik}}^1 e^{\delta_k \Omega} - {z_{ik}}^0\right]\sqrt{q_k(\Omega)}\right|^2\right)^{\frac{1}{2}}$$
$$\le \|v(t)\|^2 + \left(\sum_{k=1}^{\infty} \left|{z_{ik}}^1 e^{\delta_k \Omega} - {z_{ik}}^0\right|^2 q_k(\Omega)\right)^{\frac{1}{2}}.$$

The above inequality, Minskowski inequality is imposed. Using the obvious inequality as stated in the previous section and equation (2) in definition 2.2, we have $\|u_i(t)\|^2 = \left(\sum_{k=1}^{\infty} |u_{ik}(t)|^2\right)^{\frac{1}{2}} \le \sigma + \left(\sum_{k=1}^{\infty}\left(2|{z_{ik}}^1|^2 e^{2\delta_k \Omega} - 2|{z_{ik}}^0|^2\right) q_k(\Omega)\right)^{\frac{1}{2}} = \sigma + \left(2 \sum_{k=1}^{\infty}\left(|{z_k}^1|^2 {q_k}^*(\Omega) - |{z_k}^0|^2\ q_k(\Omega)\right)\right)^{\frac{1}{2}}$.

It has been defined that $z^0, z^1 \in \varphi_1(\Omega)$, we have

$$\|u_i(t)\|^2 = \left(\sum_{k=1}^{\infty} |u_{ik}(t)|^2\right)^{\frac{1}{2}} \le \sigma + \rho_i - \sigma = \rho_i.$$

Consequently, (12) is admissible,

Thus, the system (6) is transferred from z^0 to z^1.

The proof of the Lemma 3.2 is complete

3.1 Differential Game of Two Pursuers with Different Forms of Constraints

The present part proposes a differential game of pursuit with the case of integral and geometric constraints imposed on the player's control functions.

$$u_{ik}(t, v) = v_k(t) - \left[{z_{ik}}^1 e^{\delta_k \Omega} - {z_{ik}}^0\right] \sqrt{q_k(\Omega)}, \quad (10)$$

where

$0 < t < \Omega, \quad i = 1,2, \quad k = 1,2, \ldots$

and

$u_{ik}(t, v) = 0,\ t > \Omega, k = 1,2, \ldots$

Also, for the second Pursuer on $[0,\ \Omega]$ that satisfy integral constrain

$$u_{ik}(t, v) = v_k(t) - e^{\delta_k t}\left[{z_{ik}}^1 e^{\delta_k \Omega} - {z_{ik}}^0\right] p_k(\Omega), \quad (11)$$

where

$0 < t < \Omega, \quad i = 1,2, \quad k = 1,2, \ldots,$

and

$u_{ik}(t, v) = 0,\ t > \Omega, i = 1,2,\ k = 1,2, \ldots$

$u_i(\cdot), i = 2$ belongs to $S(\cdot)$ respectively, we have

and for the second strategy (11) to show that is admissible. Using Murkowski inequality and equation (5) of definition 2.2, we obtain

$$\left(\int_0^\Omega ||u_i(s)||^2 ds\right)^{\frac{1}{2}} = \left(\sum_{k=1}^{\infty} \int_0^\Omega |v_k(t) - e^{\delta_k s}[{z_k}^1 e^{\delta_k \Omega} - {z_k}^0] p_k(\Omega)\ |^2 ds\right)^{\frac{1}{2}}$$

$$\le \left(\int_0^\Omega ||v(s)||^2 ds\right)^{\frac{1}{2}} + \left(\sum_{k=1}^{\infty} \int_0^\Omega |e^{\delta_k s}[{z_k}^1 e^{\delta_k \Omega} - {z_k}^0] p_k(\Omega)\ |^2 ds\right)^{\frac{1}{2}}$$

$$= \sigma + \sum_{k=1}^{\infty} |{z_k}^1 e^{\delta_k \Omega} - {z_k}^0|^2 (p_k(\Omega))^2 \int_0^\Omega e^{2\delta_k s} ds.$$

Using the obvious inequality, we have

$$\left(\int_0^{\Omega}||u_i(s)||^2ds\right)^{\frac{1}{2}}$$
$$\leq \sigma + \left(\sum_{k=1}^{n}(2|z_k{}^0|^2 p_k(\Omega) + 2|z_k{}^1|^2 p_k{}^*(\Omega))\right)^{\frac{1}{2}}.$$
From the fact that $z^0, z^1 \in \varphi_2\,(\Omega)$, we obtain

$$\left(\int_0^{\Omega}||u_i(s)||^2ds\right)^{\frac{1}{2}} \leq \sigma + +\rho_i - \sigma = \rho_i.$$

And so is admissible.
Next, is to show that pursuit is completed at time Ω:
From fact that the infinite system of 1st-order differential equations (3) has the following solution
$z_{ik}(\Omega) = \beta_{ik}(\Omega)e^{-\delta_k \Omega}, k = 1,2, \dots, i = 1,2 \quad (14)$
where
$$\beta_{ik}(\Omega) = z_{ik}{}^0 - \int_0^{\Omega} e^{\delta_k s}u_{ik}(s)ds + \int_0^{\Omega} e^{\delta_k s}v_k(s)ds\ , k = 1,2, \dots, i = 1,2.$$
Then using (14) and strategy (10), we obtain
$$\beta_{ik}(\Omega) = z_{ik}{}^0 - \int_0^{\Omega} e^{\delta_k s}\left(v_k(t) - \left[z_{ik}{}^1 e^{\delta_k \Omega} - z_{ik}{}^0\right]\sqrt{q_k(\Omega)}\right)ds + \int_0^{\Omega} e^{\delta_k s}v_k(s)ds,$$
and
$$\beta_{ik}(\Omega) = z_{ik}{}^0 + \left[z_{ik}{}^1 e^{\delta_k \Omega} - z_{ik}{}^0\right]\sqrt{q_k(\Omega)}\int_0^{\Omega} e^{\delta_k s}\,ds = z_{ik}{}^1 e^{\delta_k \Omega}$$
Consequently,
$z_{ik}(\Omega) = \beta_{ik}(\Omega)e^{-\delta_k \Omega} = z_{ik}{}^1.$

Hence, pursuit is completed in differential gam$^{\Phi}{}_1$ e
Proof that pursuit is completed: Using equation (14) and strategy (11), in view of the previous cases, it is not
difficult to show that
$$\beta_{ik}(\Omega) = z_{ik}{}^0 - \int_0^{\Omega} e^{\delta_k s}\left(v_{ik}(t) - e^{\delta_k s}\left[z_{ik}{}^1 e^{\delta_k \Omega} - z_{ik}{}^0\right]\ p_k(\Omega)\right)ds + \int_0^{\Omega} e^{\delta_k s}v_k(s)ds,$$
Therefore,
$$\beta_{ik}(\Omega) = z_{ik}{}^0 + \left[z_{ik}{}^1 e^{\delta_k \Omega} - z_{ik}{}^0\right]p_k(\Omega)\int_0^{\Omega} e^{2\delta_k s}\,ds = z_{ik}{}^1 e^{\delta_k \Omega}$$
Thus,
$z_{ik}(\Omega) = \beta_{ik}(\Omega)e^{-\delta_k \Omega} = z_{ik}{}^1.$

Hence, in differential game $^{\Phi}{}_2$ pursuit can be completed. The proof of theorem 4.1 is completed.

4.0 Conclusions

We have studied a differential game of pursuit's problem with two Pursuers chasing one Evader and with differential forms of constraints (i.e., geometric and integral constraint). The control functions of first Pursuer satisfied the geometric constraint and that of the second Pursuer satisfied the integral constraint. The game was described by infinite system of first-order differential equations in l_2 space.
We have solved a control problem of single player of bringing the state z^0 of (6) to z^1 for a finite time. Moreover, we give conditions of completion of pursuit for each case of constraints in differential game $^{\Phi}{}_1$ and $^{\Phi}{}_2$ respectively. In addition to this, the strategy of each Pursuer was constructed in an explicit form.

Declarations

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Not Applicable

Consent for publication
All authors have read and consented to the publication of the manuscript.

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All authors declare no competing interests.

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